Extending the Standard Model with a Fermionic WIMP Dark Matter Candidate and Quintessence

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Abstract

This work explores an extension of the Standard Model (SM) Lagrangian to incorporate a candidate for dark matter and a mechanism for dark energy. Dark matter is introduced as a fermionic Weakly Interacting Massive Particle (WIMP), a Dirac fermion denoted by χ, which interacts with the SM sector via a Yukawa coupling to the Higgs field. Dark energy is modeled by a quintessence scalar field, Q, with a canonical kinetic term and a general potential $V(Q)$. We present the extended Lagrangian and verify its gauge invariance under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. This framework provides a minimal extension to address two of the most significant observational puzzles in modern cosmology.

1 Introduction

The Standard Model of particle physics, despite its remarkable success in describing fundamental particles and their interactions, fails to account for several key cosmological observations. Among these are the existence of dark matter, which constitutes the majority of matter in the universe, and dark energy, which drives the current accelerated expansion of the universe. This paper proposes a minimal extension to the SM Lagrangian to incorporate candidates for both dark matter and dark energy.

We model dark matter as a fermionic WIMP, a well-motivated paradigm that naturally arises in many extensions of the SM. For dark energy, we employ a quintessence model, where a slowly rolling scalar field drives cosmic acceleration.

2 The Extended Lagrangian

The total Lagrangian, $\mathcal{L}_{\text{total}}$, is given by the sum of the Standard Model Lagrangian, $\mathcal{L}_{\text{SM}}$, a dark matter Lagrangian, $\mathcal{L}_{\text{DM}}$, and a dark energy Lagrangian, $\mathcal{L}_{\text{DE}}$:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{DE}}$$

2.1 Dark Matter Sector

The dark matter candidate is a Dirac fermion, χ, which is assumed to be a singlet under the $SU(3)_C$ and $SU(2)_L$ gauge groups, but carries a $U(1)_Y$ hypercharge q_χ. Its Lagrangian is given by:

$$\mathcal{L}_{\text{DM}} = \bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi - y_\chi \bar{\chi}\tilde{\phi}\chi$$

where:

- $\bar{\chi}$ is the Dirac adjoint of χ.
- γ^μ are the Dirac gamma matrices.
- $D_\mu = \partial_\mu - ig_1 q_\chi B_\mu$ is the covariant derivative for χ, with g_1 being the $U(1)_Y$ coupling constant and B_μ the $U(1)_Y$ gauge field. We set $q_\chi = 1/2$ for gauge invariance of the Yukawa term.
- m_χ is the mass of the dark matter fermion.
- y_χ is the Yukawa coupling constant.
- $\tilde{\phi} = i\sigma_2\phi^*$ is the charge conjugate of the Standard Model Higgs doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, where σ_2 is the second Pauli matrix. The term $\bar{\chi}\tilde{\phi}\chi$ ensures an $SU(2)_L \times U(1)_Y$ invariant interaction that generates a mass for χ after electroweak symmetry breaking.

2.2 Dark Energy Sector (Quintessence)

Dark energy is modeled by a quintessence scalar field, Q, which is a singlet under all Standard Model gauge groups. Its Lagrangian is:

$$\mathcal{L}_{\text{DE}} = \frac{1}{2}(\partial_\mu Q)(\partial^\mu Q) - V(Q)$$

where $V(Q)$ is the potential for the quintessence field. The specific form of $V(Q)$ (e.g., exponential, power-law, or tracking potentials) determines the cosmological evolution and equation of state of dark energy. For this work, we keep $V(Q)$ general.

3 Gauge Invariance

The extended Lagrangian is constructed to be invariant under the Standard Model gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$.

- $\mathcal{L}_{\text{SM}}$: Is invariant by construction.
- $\mathcal{L}_{\text{DM}}$:
  - The kinetic term $\bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi$ is invariant due to the covariant derivative D_μ.
  - The Yukawa term $-y_\chi \bar{\chi}\tilde{\phi}\chi$ is invariant. χ transforms under $U(1)_Y$ with charge q_χ. $\tilde{\phi}$ transforms in the conjugate representation of ϕ under $SU(2)_L$ and $U(1)_Y$. With $q_\chi = 1/2$, the combination is gauge invariant. Since χ is an $SU(3)_C$ and $SU(2)_L$ singlet, these symmetries are trivially preserved.
- $\mathcal{L}_{\text{DE}}$: Since Q is a gauge singlet, its Lagrangian is manifestly invariant under G_{SM}.

Thus, the total Lagrangian $\mathcal{L}_{\text{total}}$ respects the gauge symmetries of the Standard Model.

4 Discussion and Outlook

We have presented a minimal extension of the Standard Model Lagrangian that incorporates a fermionic WIMP dark matter candidate and a quintessence scalar field for dark energy. The model is gauge invariant under the Standard Model gauge group.

Further work is required to explore the phenomenological implications of this model. This includes:

- Specifying the quintessence potential $V(Q)$ and analyzing the resulting cosmological dynamics. [Specific Quintessence Potential discussion would go here].
- Calculating dark matter relic abundance and direct/indirect detection cross-sections for the WIMP candidate χ. This will depend on the parameters m_χ and y_χ, as well as interactions arising from potential portals beyond the Higgs.

- Constraining the model parameters using current and future experimental data from collider experiments, dark matter searches, and cosmological observations. [Further Phenomenological Analysis would go here].

Despite its simplicity, this framework provides a self-consistent starting point for addressing the dark matter and dark energy puzzles within a particle physics context.

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References

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